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On the identification of the nuclear forward glory in heavy-ion scattering

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Abstract. We present a simple method which enables us to identify the occurence of a forward glory in heavy ion scattering data. The method is successfully applied to the elastic scattering data of ^{12}C (65 MeV), ^{13}C (60 MeV), ^{15}N (85 MeV) and ^{16}O (75 MeV) on ^{28}Si.

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The determination of the near-forward nuclear amplitude from the experimental data, in a model independent way, is an interesting current subject in nucleus-nucleus collisions because it allows to identify the occurence of glory scattering, a characteristic feature of surface refraction. This nuclear amplitude has been obtained from the "sum of difference" (SOD) method which is based on a modified version [1] of the generalized optical theorem [2], [3]. As "experimental input", the SOD method uses the "sum of difference" cross section $\sigma_{\mathrm{SOD}}(\theta)$ given by the integral of the difference between the measured elastic cross-section $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$ and the Rutherford one $\frac{\mathrm{d}\sigma_{\mathrm{R}}}{\mathrm{d}\Omega}$, in the angular range (θ, π). As $\sigma_{\mathrm{SOD}}(\theta)$ should cover the near forward direction, the method requires the measure of $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$ in almost the whole angular range. In addition, as $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$ is known only at a sequence of discrete angles, interpolation is necessary to evaluate the integral.

These being, the purpose of this paper is to present a method, alternative to the SOD method, in which the nuclear amplitude is obtained directly from the knowledge of $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$ in the restricted angular range where the forward glory takes place. So, let us consider the elastic scattering between two nuclei and write $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$ in the form:

$$\frac{d\sigma}{d\Omega} = |f_n(\theta)|^2 + \frac{d\sigma_R}{d\Omega} + 2|f_n(\theta)|\sqrt{\frac{d\sigma_R}{d\Omega}} \cos(\phi_n - \phi_c) \quad (1)$$

where $\mathrm{f_n}(\theta)$ and ϕ_{n} are usually called the nuclear amplitude and the nuclear phase respectively [1], [4] although they retain of course a coulomb potential dependance. The phase ϕ_{c} is the coulomb phase given by

$$\phi_c = \pi - 2\eta \ln(\sin\frac{\theta}{2}) + 2\sigma_0 \quad (2)$$

σ_0 and $\eta = \frac{\mathrm{Z_1 Z_2 e^2}}{\hbar \mathrm{v}}$ are respectively the s-wave coulomb phase shift and the Sommerfeld parameter.

For large η, the angular range where glory effects occur, is such that $|\mathrm{f_n}(\theta)|^2$ is negligibly small compared with both $\frac{\mathrm{d}\sigma_{\mathrm{R}}}{\mathrm{d}\Omega}$ and the interference term in (1). So, ignoring $|\mathrm{f_n}(\theta)|^2$, the expression (1) can be rewritten in the form:

$$\frac{1}{2}\sqrt{\frac{d\sigma_R}{d\Omega}}(\frac{d\sigma}{d\sigma_R} - 1) = |f_n(\theta)| \cos(\phi_n - \phi_c) \quad (3)$$

which is expected to be accurate for $\eta \gg 1$ and small scattering angles ($\theta \ll \theta$ rainbow).

So, the left hand side of (3) is the "experimental input" of the present method. In the near forward direction, the coulomb phase (2) varies rapidly and therefore the quantity $\frac{1}{2}\sqrt{\frac{\mathrm{d}\sigma_{\mathrm{R}}}{\mathrm{d}\Omega}}(\frac{\mathrm{d}\sigma}{\mathrm{d}\sigma_{\mathrm{R}}} - 1)$ oscillates with spacing $\delta \simeq \frac{\pi\theta}{\eta}$ and envelopes given by $|\mathrm{f_n}(\theta)|$.

According to the semi-classical picture [5], these envelopes should behave like the Bessel function $\mathrm{J_0}(\mathrm{l_g}\theta)$ where $\mathrm{l_g}$ is the glory angular momentum, if a nuclear glory occurs. As we shall see, such envelopes are easy to draw providing $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$ is measured with sufficient accuracy and angular resolution.

Before going into the application of the present method to recent experimental data, let us first test the accuracy of the approximate expression (3). To do that, we consider the scattering of two nuclei and take for the left hand side of (3) the numerical results of an optical model calculation which fits the existing data. Then one can compare the envelope of the oscillations given by $\frac{1}{2}\sqrt{\frac{\mathrm{d}\sigma_{\mathrm{R}}}{\mathrm{d}\Omega}}(\frac{\mathrm{d}\sigma}{\mathrm{d}\sigma_{\mathrm{R}}} - 1)$ with the calculated curve $|\mathrm{f_n}(\theta)|$. Further, to draw these envelopes, the choice of the colliding nuclei as well as of the incident energy, should be such that the period of the

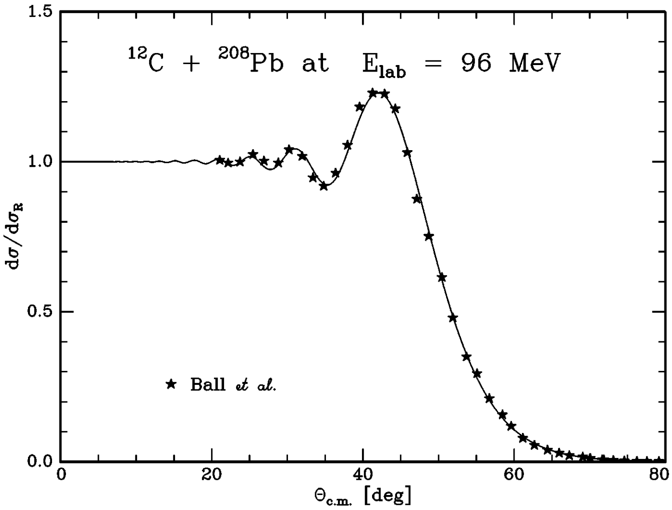

Fig. 1. Angular distribution of elastic scattering of ^{12}C on ^{208}Pb at $E_L = 96$ MeV, from [6]. The solid line indicates the results of the optical model calculation

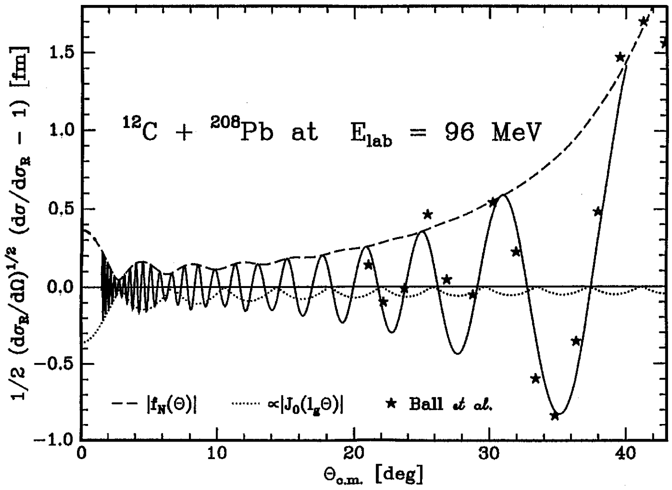

Fig. 2. Elastic scattering of ^{12}C on ^{208}Pb. Calculated values : $\frac{1}{2}\sqrt{\frac{d\sigma_R}{d\Omega}}(\frac{d\sigma}{d\sigma_R}-1)$ (solid line), $|f_n(\theta)|$ (broken line) and $|f_n(0)|\,|J_0(l_g\theta)|$ (dashed line). The points have been obtained taking for $\frac{d\sigma}{d\Omega}$ the experimental values of Fig. 1 (see text)

fast oscillations, $\delta \simeq \frac{\pi\theta}{\eta}$, is small compared with the period of the glory oscillations, $\delta_g \simeq \frac{\pi}{l_g}$, in the small angle region. The angle of the first glory minimum is given by $\theta_{mi} \simeq \frac{3\pi}{4l_g}$. So, one can reasonably ask that $\delta \ll \delta_g$ for $\theta \simeq \theta_{mi}$. This gives $\eta \gg (\sim 2)$ which is consistent with the condition for neglecting $|f_n(\theta)|^2$ in (1).

So, let us consider the elastic scattering of ^{12}C on ^{208}Pb at E = 96MeV. Figure 1 shows the experimental data as well as the optical model fit obtained with a Woods-Saxon potential whose parameters are [6]:

$$V_0 = 40\text{MeV}, W_0 = 25\text{MeV},$$
$$r_v = r_w = 1.201\text{fm}, a_v = a_w = .664\text{fm} \qquad (4)$$

This example, for which $\eta = 27.4$, has been selected for its didactic value [7] since the effect we want to discuss is clearly evidenced in the calculations even if the fast oscillations given by (3) may be hard to resolve experimentaly. The calculated values of $\frac{1}{2}\sqrt{\frac{d\sigma_R}{d\Omega}}(\frac{d\sigma}{d\sigma_R}-1)$ using for the θ-axis a linear and a log. scale, are shown by solid lines in Figures 2 and 3 respectively. The calculated values of $|f_n(\theta)|$ are shown by the broken lines. The dashed line is a plot of $|f_n(0)|\,|J_0(l_g\theta)|$ using for l_g the value $l_g = 47$ obtained from the classical deflection function calculated with the real part of the potential (4). As seen in Fig. 2, the "exact" curve $|f_n(\theta)|$ represents the envelope of the oscillations given by the left hand side of the approximate expression (3) over a very broad angular range outside the near forward direction. The points have been obtained taking for $\frac{d\sigma}{d\Omega}$ the experimental values of Fig. 1.

Figure 3 allows us to analyse the small angle region in more detail. The comparaison between the two curves $|f_n(\theta)|$ and $|f_n(0)|\,|J_0(l_g\theta)|$ in the angular range $\theta \leq 10°$ allows us to conclude unambigously that the Woods-Saxon potential (4) gives rise to a net forward glory.

Let us now consider the application of the method to the experimental data [8] obtained in the elastic scattering

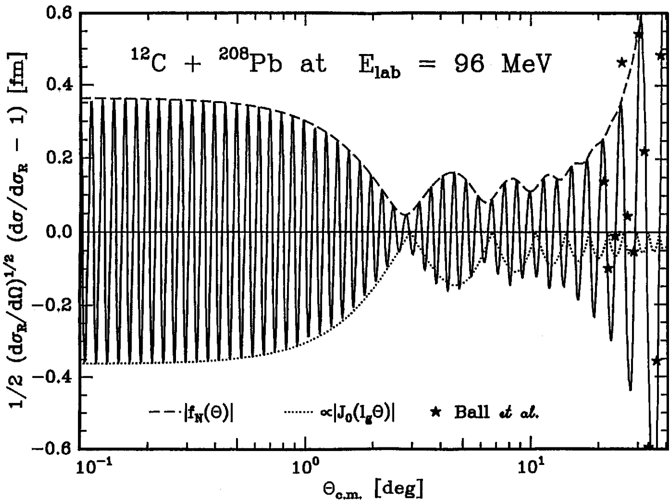

Fig. 3. The same as for Fig. 2, using a log. scale for the θ-axis

of ^{12}C (65 MeV), ^{13}C (60 MeV), ^{15}N (85 MeV) and ^{16}O (75 MeV) on ^{28}Si. An example of the measured ratio $\frac{d\sigma}{d\sigma_R}$ is shown in Fig. 4 for $^{12}C + ^{28}Si$ (65 MeV). The points in the Figures 5, 6, 7 and 8 are the plot of $\frac{1}{2}\sqrt{\frac{d\sigma_R}{d\Omega}}(\frac{d\sigma}{d\sigma_R}-1)$ using for $\frac{d\sigma}{d\Omega}$ the measured values. In order to analyse these plots, we have first drawn the broken curves through the points that allows us to localize approximatively the extrema of the fast oscillations. The period of these oscillations is quite accurately given by $\delta \simeq \frac{\pi\theta}{\eta}$. One immediately remarks that, in all the cases, the way in which the amplitude of the successive extrema varies, is clearly reminiscent of a glory pattern. Such a behavior emerges even more clearly by finding two parameters $|f_n(0)|$ and l_g such that $|f_n(0)|\,|J_0(l_g\theta)|$ envelopes the fast oscillations. Moreover, as a pure $J_0(l_g\theta)$ behavior should hold only at very small angles, these parameters have been obtained by

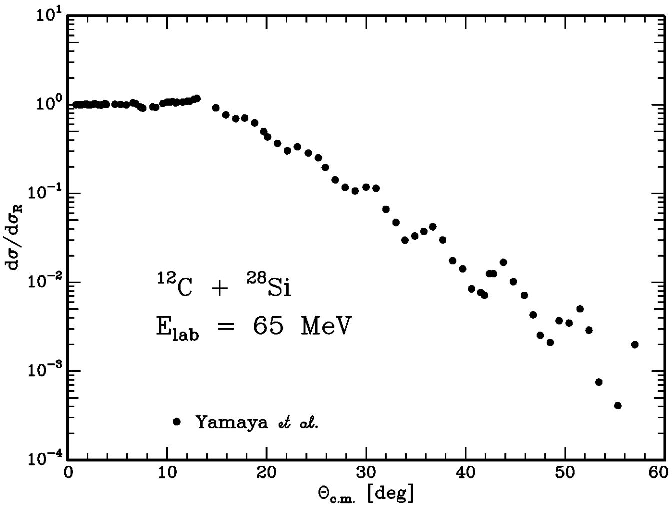

Fig. 4. Elastic scattering angular distribution of ^{12}C on ^{28}Si at $E_L = 65$ MeV, from [8]

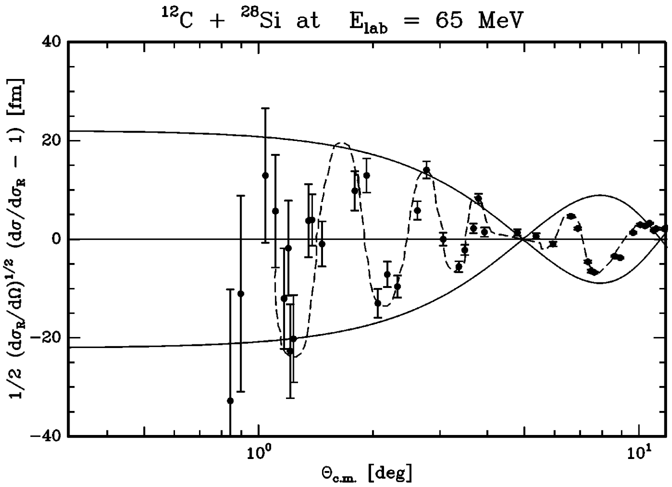

Fig. 5. Elastic scattering of ^{12}C on ^{28}Si ($E_L = 65$ MeV). The points are the plot of $\frac{1}{2}\sqrt{\frac{d\sigma_R}{d\Omega}}(\frac{d\sigma}{d\sigma_R} - 1)$ as a function of θ (log. scale) using for $\frac{d\sigma}{d\Omega}$ the experimental data. The broken line has been drawn through the points to localize the extrema of the fast oscillations. The solid line has been obtained by searching the parameters $|f_n(0)|$ and l_g such that $|f_n(0)||J_0(l_g\theta)|$ envelopes the rapid oscillations (see text)

taking into account only the extrema for which the envelope corresponds to the forward glory maximum, i.e. those contained in the angular range $\theta \leq 4°$, for all the cases.

The result one obtains is given by the solid curve, with $|f_n(0)|$ and l_g given in Table I together with the values $(|f_n(0)|, l_g)_{SOD}$ obtained in reference [8] using the "sum of difference" method. This curve shows clearly the pronounced forward rise as well as the first maximum of the J_0 Bessel function, i.e. the fingerprint of the glory effect, reflecting the importance of surface refraction.

The difference in the $(|f_n(0)|, l_g)$ values (reported in Table I) obtained with the present method and with the SOD-method appeals two remarks:

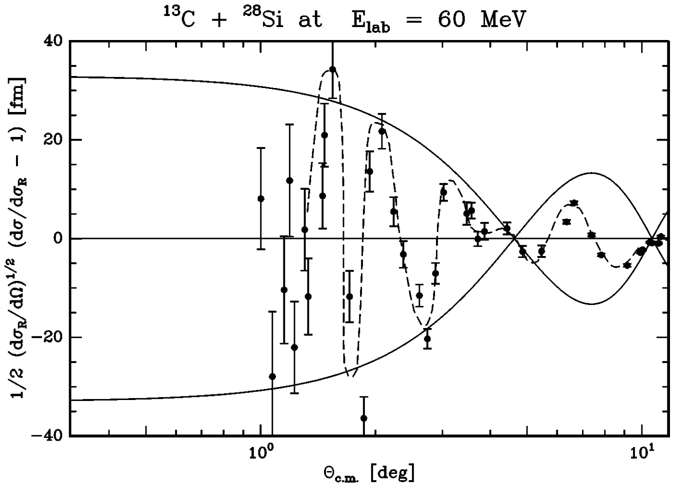

Fig. 6. Same as for Fig. 5 for the elastic scattering of ^{13}C on ^{28}Si ($E_L = 60$ MeV)

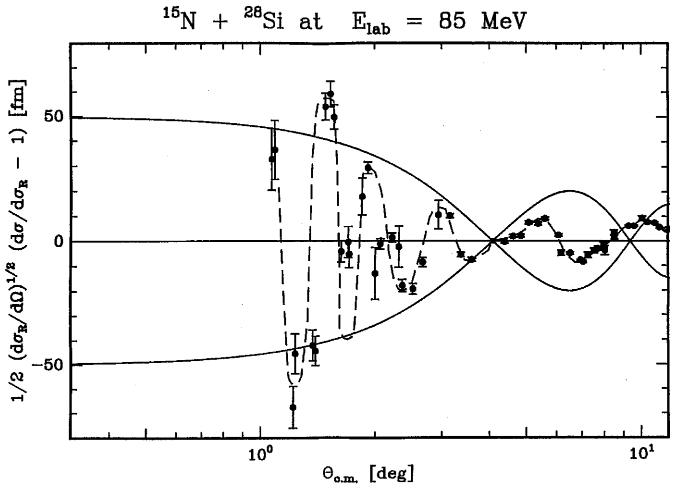

Fig. 7. Same as for Fig. 5 for the elastic scattering of ^{15}N on ^{28}Si ($E_L = 85$ MeV)

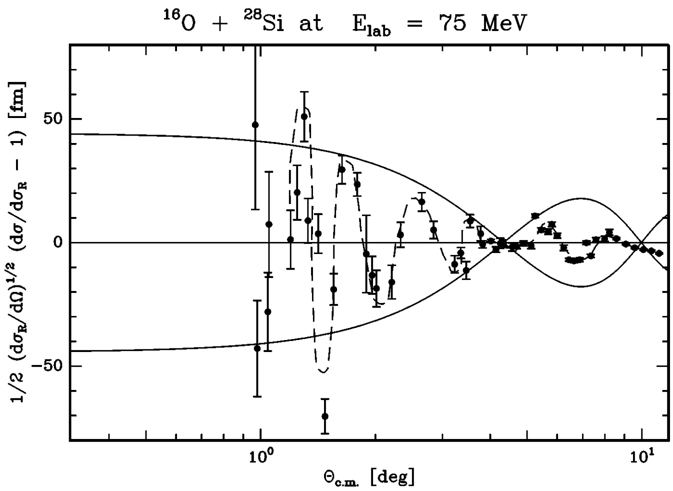

Fig. 8. Same as for Fig. 5 for the elastic scattering of ^{16}O on ^{28}Si ($E_L = 75$ MeV)

Table 1. Numerical results obtained for $|f_n(0)|$ and l_g.The values $|f_n(0)|$(SOD) and l_g(SOD) are also reported, see reference [8]

projectile	E_L (MeV)	η	$\|f_n(0)\|$ (fm)	l_g	$\|f_n(0)\|$ (SOD)	l_g (SOD)
^{12}C	65	5.68	22	28	10	30
^{13}C	60	6.16	33	30	21	30
^{15}N	85	6.26	50	34	48	40
^{16}O	75	8.14	43	32	35	45

i) Both methods require the drawing of the envelope of very fast oscillations, but these oscillations are obtained from the elastic data in a quite different way. In the SOD method, each value of $\sigma_{\rm SOD}(\theta)$ results from the integration of $d\sigma/d\Omega$ in the range (θ, π) and requires in addition an interpolation procedure [see ref [8]]. In the present method, the fast oscillations are given by the expressions (3) in which the experimental input is the measured $d\sigma/d\Omega$ itself.

ii) At present, the only available data in the most near forward direction are those of ref [8]. More accurate measurements are therefore required in this angular range to improve the determination of $|f_n(0)|$.

In summary, the forward glory scattering phenomenon is well identified in the reported heavy ion scattering data. This has be done by using the angular dependence of the quantity $\frac{1}{2}\sqrt{\frac{d\sigma_R}{d\Omega}}(\frac{d\sigma}{d\sigma_R} - 1)$ which requires the knowledge of the scattering cross-section in the near forward angular range.

It would be interesting to have more experimental data at near forward angles particularly with neutron halo nuclei as this has been suggested by several authors [9]. In fact, the particular behavior of scattering with a neutron halo nucleus is related to the extended matter distribution of the halo which stretch the range of the effective nuclear force to a long distance as well as to deeper absorption due to the weak binding energy of the neutrons in the halo.

The identification of a forward nuclear glory and the determination of the nuclear amplitude $|f_n(\theta)|$ could help to understand how the refractive effects due to the extension of the nuclear force and to the absorption (imaginary part of the potential) are playing a special role in the case of neutron halo nuclei.

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